Algorithms for the Generation of Daubechies Orthogonal Least Asymmetric Wavelets and the Computation of their Holder Regularity

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Abstract

Explicit algorithms are presented for the generation of Daubechies compact orthogonal least asymmetric wavelet filter coefficients and the computation of their Holder regularity. The algorithms yield results for any number N of vanishing moments for the wavelets. These results extend beyond order $N = 10$ those produced by Daubechies for the values of the filter coefficients and those produced by Rioul for the values of their Holder regularity. Moreover, they reveal that the choice of phase for the filters published by Daubechies for orders $N = 4$ to $N = 10$ was not made consistently. In particular, her filter coefficients for orders $N = 7$ to $N = 9$ should be reflected to their mirror image sequence.

1 Introduction

Daubechies discovery of compact orthogonal wavelets [1] remains one of the most important contributions responsible for the growth of the field of wavelet analysis over the past decade. Using the same general approach applied to the original family of wavelets known as "minimum phase", Daubechies later added variations yielding other families such as the "least asymmetric" family [2, 3]. This particular family was used as the base upon which were built the interval wavelets of Cohen, Daubechies, and Vial [4].

Although mathematical principles are expounded in these papers, none provide explicit computational algorithms. In particular, constructing the least asymmetric wavelets requires selecting roots of a polynomial. Unless this selection is automated by an explicit algorithm, it is possible that choices may not be made consistently in the same manner. In this report, I present explicit algorithms for the generation of Daubechies orthogonal least asymmetric wavelets and computation of their Holder regularity. The algorithms are applicable to any order N for the number of vanishing moments of the wavelets.

2 Methods

Wavelet filter coefficients were generated by combining methods for the construction of the Lagrange à trous filter polynomials defined by Shensa [5], their spectral factorization to alternative square root factors characterized by their total phase non-linearity as explained by Daubechies [3, 2], labelling the roots of these factors with a binary code, generating the possible combinatorial subsets for these binary codes as explained by Reinhard et al [6], searching through the subsets of roots to select the one with the minimum total phase non-linearity, then comparing the coefficient sequence from the selected subset of roots to its mirror image reflected sequence, and finally selecting that root subset and filter sequence which has

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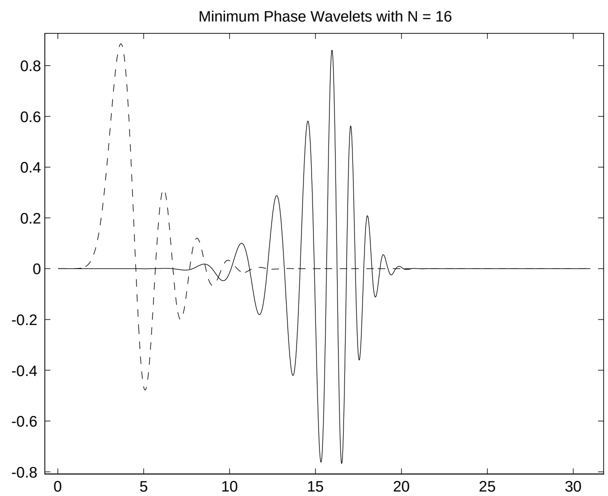

Figure 1: Minimum phase scaling (dashed line) and wavelet (solid line) functions with $N = 16$ vanishing moments.

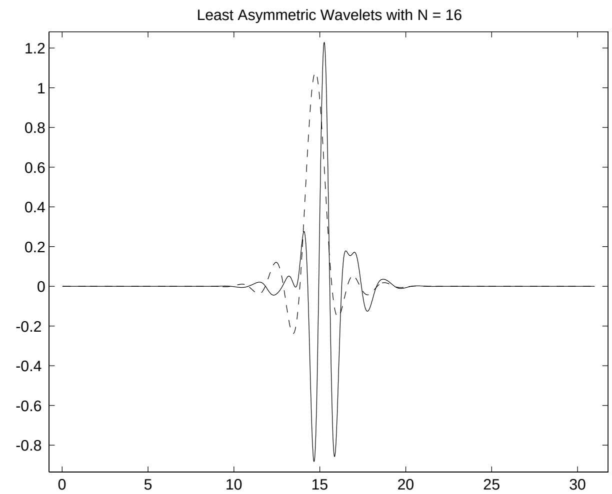

Figure 2: Least asymmetric scaling (dashed line) and wavelet (solid line) functions with $N = 16$ vanishing moments.

both the minimum total phase non-linearity, and of the two complementary sequences with the same total phase non-linearity, the one with the minimum total phase delay. Holder regularity of these filter sequences was computed by a modification of the method of Rioul [7, 8]. Complete details of the algorithms will be provided in the final version of this report.

3 Results

Figures 1 and 2 display scaling (dashed lines) and wavelet (solid lines) functions corresponding to the scaling and wavelet filter sequences for the minimum phase and least asymmetric wavelets of order $N = 16$ with 32 coefficients and support interval of length 32. Figure 3 displays a plot of the Holder regularity α versus the wavelet order N for the minimum phase (dashed line) and least asymmetric (solid line) wavelets. Asymptotically, $\alpha \approx N/4$ appears to hold true. Comparison of actual filter coefficient sequences for orders $N = 4$ to $N = 10$ of the least asymmetric wavelets with the values published by Daubechies [3]

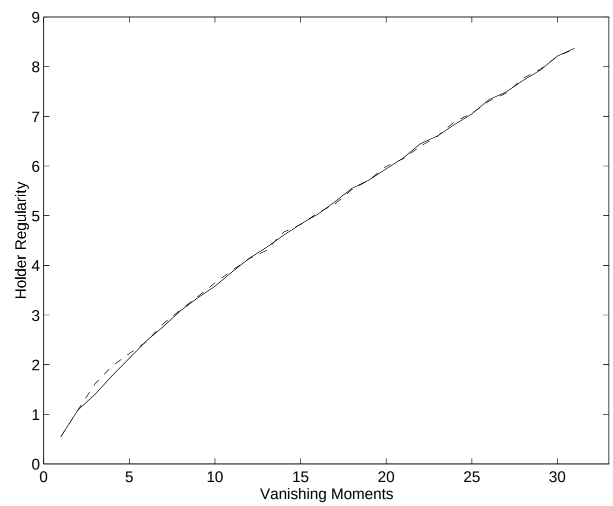

Figure 3: Holder regularity α versus number N of vanishing moments for minimum phase (dashed line) and least asymmetric (solid line) wavelets.

reveals that she chose the complementary filter for orders $N = 7$ to $N = 9$. This choice is not the one with the minimum total phase delay of the two complementary choices which have total phase non-linearity equal to the minimum.

4 Discussion

Explicit computational algorithms have been developed for generating Daubechies compact orthogonal least asymmetric wavelet filter coefficients and computing their Holder regularity. These automated algorithms are valid for any order N of wavelet and insure that the same consistent choice of roots is always made in the computation of the filter coefficients. They have been used here to generate all least asymmetric wavelet filters up to length 64 with order $N = 32$ and Holder regularity α as displayed in Figure 3. Comparison of the generated coefficients with Daubechies published table [3] revealed that an inconsistent choice was made in her table for order $N = 7$ to $N = 9$. Similar and/or related inconsistencies may explain the difficulty some readers may have experienced (as the author has) in using tables of coefficients for boundary wavelets together with the interior wavelets for the least asymmetric family [4]. The importance of preventing such difficulties justifies the necessity of clarifying explicit algorithms for the computation of filter coefficients.

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